# EEE4118F Controller Design Report


Presented By:

Junior Makgoe (MKGPUL005)

Mark Njoroge (NJRMAR003)

## Introduction

Following the system Identification, this report focuses on the controller design for the various plant models obtained. The controller was designed with the goal of achieving maximum performance on a nominal plant as well as maintaining stability on the other different models. Quantitative Feedback Theory was used to achieve Robust Stability.

## Methodology

### Summary of Plant and Specifications required

The plant models obtained from system identification is $P = \frac{A}{\tau s + 1}$ where A =[78.195, 88.742, 16.52, 17.26, 77.0,5 83.8375] and $\tau$ = [1.311, 1.661, 0.161, 0.121, 8.75, 10.34]. The 6 different plants represent the different conditions the motor is put under. The controller was designed to adhere to the following user specifications:

| **Specification** | **Value** |
|---|---|
| Speed Controller Response | 1.2s |
| Overshoot in velocity loop | 25% |
| Margin for digital design | 10° |
| Unsaturated Step size | 10° |
| Overshoot in Position loop | 12.5% |

*Table 1: User Specifications*

## Design Background

The plant models obtained were for the velocity of the motor, however we wish to control the position of the motor. There are various methods that can achieve this and we chose to implement a cascaded control loop. The block diagram for the whole system is shown in the figure below:

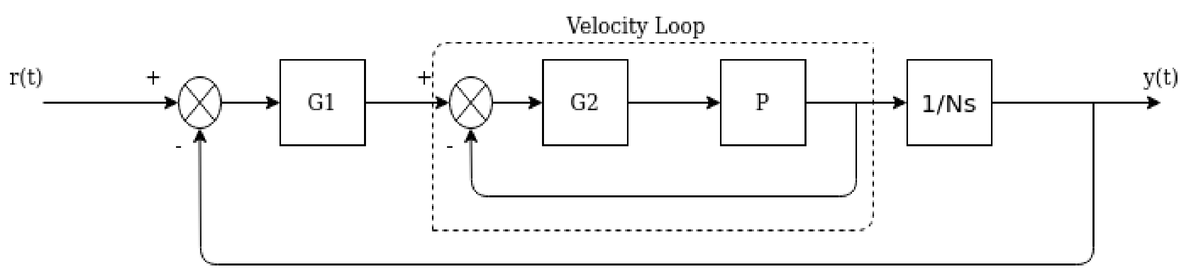


*Figure 1: Block Diagram For Servo Motor*

The inner loop will first be used to control the speed of the motor. Once that is achieved, the velocity loop is treated as a single block and then $G_1$ is designed to control the position of the motor. The ratio between the motor shaft and the position indicator is also included and put in the same block as the integrator that converts the angular velocity to position.

In addition, because we are implementing a digital controller, digital design techniques need to be used. However, since we are familiar with linear design, we take into consideration the sampling effect by including the right-hand plane zero. With this done we can design freely using the methods we have learnt.

Furthermore, the 10° margin for digital design needs to be considered. This specification is met by making sure there is an additional 10° between the frequencies of design and their respective bounds in the INC. This would allow for the uncertainties in the digital implementation of the controllers.

## Designing Velocity Control

The time domain response specifications provided were used to design the controller by converting them into bounds in the frequency domain that were to be plotted onto the inverse Nichols chart. Because we are trying to minimise output disturbance, we are dealing with $\left|\frac{1}{1+L}\right|$ and that can be modelled by $e(t) = r(t) - y(t)$ which in the frequency domain is a high pass filter. The filter can therefore be described by the characteristic equation and the parameters are found from the user requirements. The following series of equations shows how the high pass filter was found.

Finding ς:
$$\%OS = e^{-\frac{\pi\varsigma}{\sqrt{1-\zeta^2}}} \tag{1}$$

$$\therefore \quad \zeta = \sqrt{\frac{(ln\%OS)^2}{(ln\%OS)^2+\pi^2}}$$

Subbing in the desired overshoot of 25%, we get $\varsigma = 0.4037$

Using this and the desired time constant of 1.2s we can get $\omega_n$ as follows:

$$\omega_n = \frac{1}{\tau\zeta} = 2.064$$

Finally, the Equation is described as:

$$T_{high} = \frac{s(\frac{2\zeta}{\omega_n})(\frac{s}{2\zeta\omega_n}+1)}{(\frac{s}{\omega_n})^2+\frac{2\zeta s}{\omega_n}+1} = \frac{0.391s(0.6s+1)}{0.235s^2+0.391s+1} \tag{2}$$

Plotting the magnitude of this function, we can find the bounds for specific frequencies required to be plotted on the INC in order to design the controller. This graph is shown below:

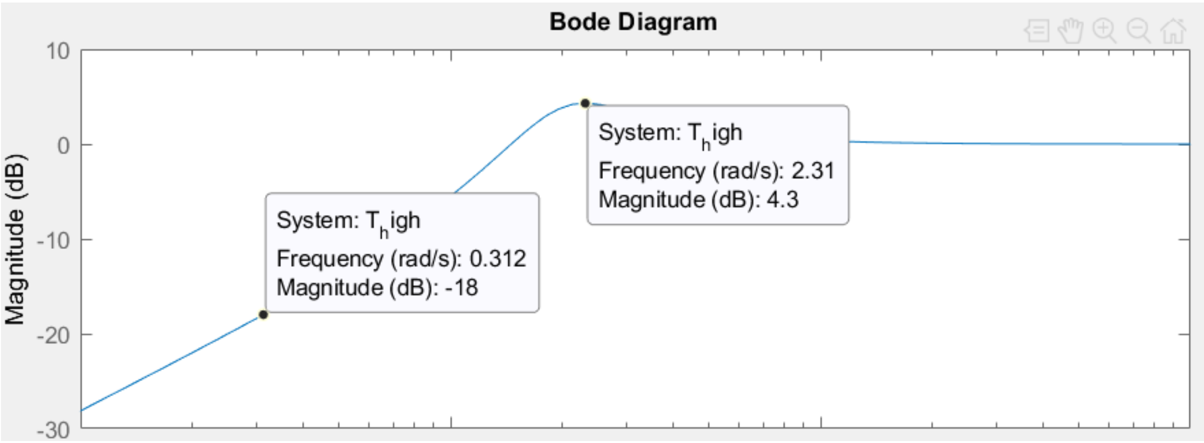

*Figure 2: Finding Bounds*

The two design bounds are therefore:

$$\left|\frac{1}{1+L}\right| < \ 4.3dB \ \forall\, \omega \quad \& \qquad \left|\frac{1}{1+L}\right| <- \ 18\ dB\ @\ \omega \leq 0.312$$

With these regions plotted in the INC, the certain test frequencies are moved around (which constitutes changing the gain and adding/removing poles and zeros in the controller) such that they do not enter the regions.

## Designing Position Control

As stated earlier, designing for position control considers the velocity loop as a single block. It also incorporates the digitisation effect from the sampling as well as the ratio between the motor shaft. All these need to be combined in order to produce a new plant that we can design a controller for.

Equation 3 details how the new plants can be calculated. We first need to express the velocity loop as a closed loop response and multiply that by the discretization effect which is a right-hand plane zero at half the sampling frequency. We also integrate the velocity so that we get a position output in the loop.

$$P_{new} = \frac{L}{1+L} \times \frac{1-\frac{sT}{2}}{30s} \tag{3}$$

where, $L \ = \ P \times G_2$

As with the velocity control, we take the time domain specifications and convert them to frequency domain variables that describe a high pass filter. The only given specification is that of overshoot, and so we choose a response time that is slower than the velocity's response time. We choose $\tau \ = \ 2s$ and with %OS = 12.5%, using the same working as before, we achieve $\zeta \ = \ 0.552$ and $\omega_n \ = \ 0.9$.

Therefore, the filter equation is as follows:

$$T_{high} = \frac{1.227s(s+1)}{1.235s^2+1.227s+1} \tag{4}$$

The magnitude of this function is plotted below:

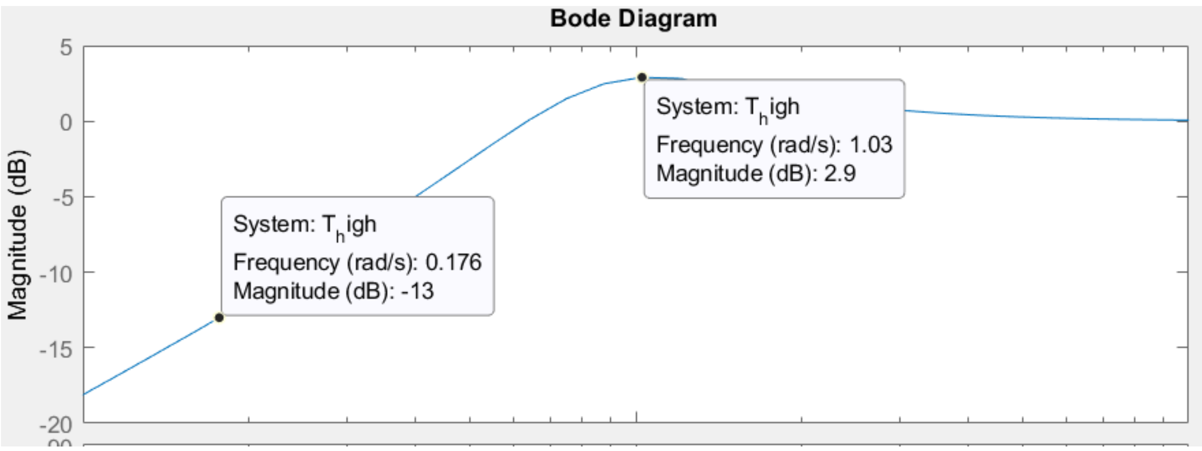

*Figure 3: Position Control High Pass Filter*

The design requirements are therefore:

$$\left|\frac{1}{1+L}\right| < \ 2.9dB \ \ \forall \omega \quad \& \qquad \left|\frac{1}{1+L}\right| <- \ 13 \ dB \ @ \ \omega \leq 0.176$$

# Results

## Velocity design results

To design for velocity control, the 6 different plant templates, with the controller G(s)=1, were plotted on the inverse Nichols chart to map out their stay out regions. The plants that were used are

$$P_1(s) = \frac{78.195}{1.311s+1} \quad P_2(s) = \frac{88.742}{1.661s+1}$$
$$P_3(s) = \frac{16.52}{0.161s+1} \quad P_4(s) = \frac{17.26}{0.121s+1}$$
$$P_5(s) = \frac{77.05}{8.75s+1} \quad P_6(s) = \frac{83.8375}{10.34s+1} \tag{5}$$

Each of these plants were evaluated at a set of discrete frequencies
W = 0.311, 1, 2, 4, 5, 7, 10, 15, 20, 30, 40, 50, 75, 100, 100, 200, 500, 1000. The template is then moved around in the Nichols chart until all boundaries mapped out by the templates are not violated. The result is shown in figure 4 below.

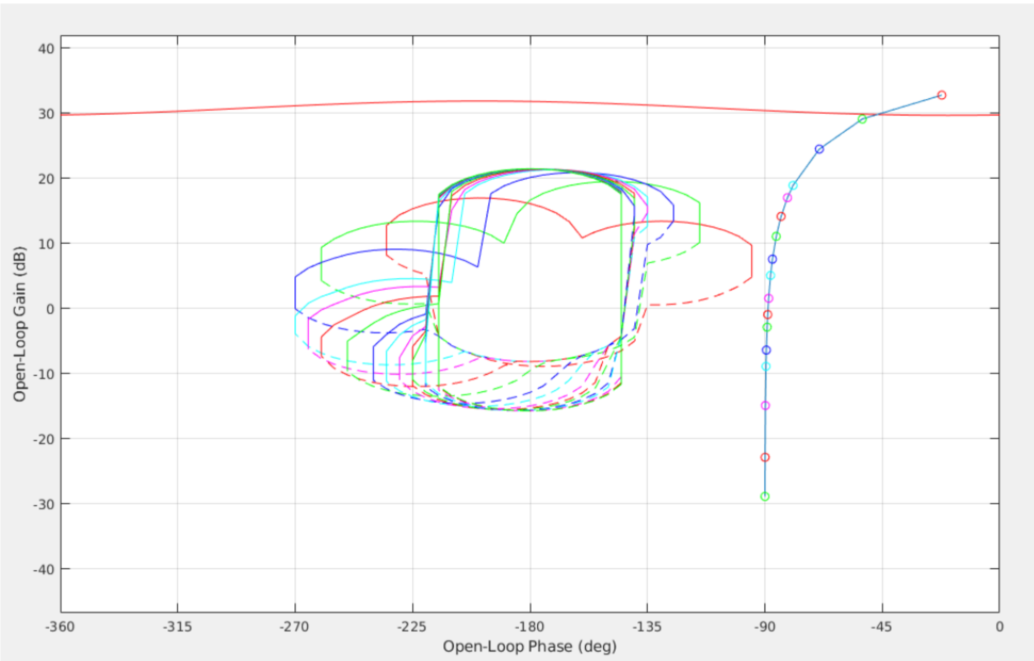


*Figure 4: Inverse Nichols chart for velocity design*

The boundary constraints are met for all values of w. As a result, the velocity control can be found by using only a proportional controller. In our case, we reduced the gain to k = 0.6018. The reason for the reduction in gain is that many iterations were done for the position control and it was found that reducing the gain to this value made the position loop control simpler.

Once the controller has been found, it's effects were simulated using the block diagram shown in figure 5. The subscript gg denotes the controller $G_2(s)$and$P(1, 1, 1)$denotes the nominal plant.

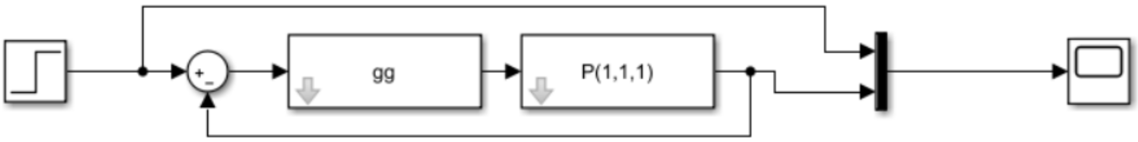


*Figure 5: Simulink block diagram for velocity control*

Using $G_2(s) = 0.6018$ and $P_1(s) = \frac{78.195}{1.311s+1}$ as our nominal plant, the step response of the velocity loop that was shown figure 1 is shown below.

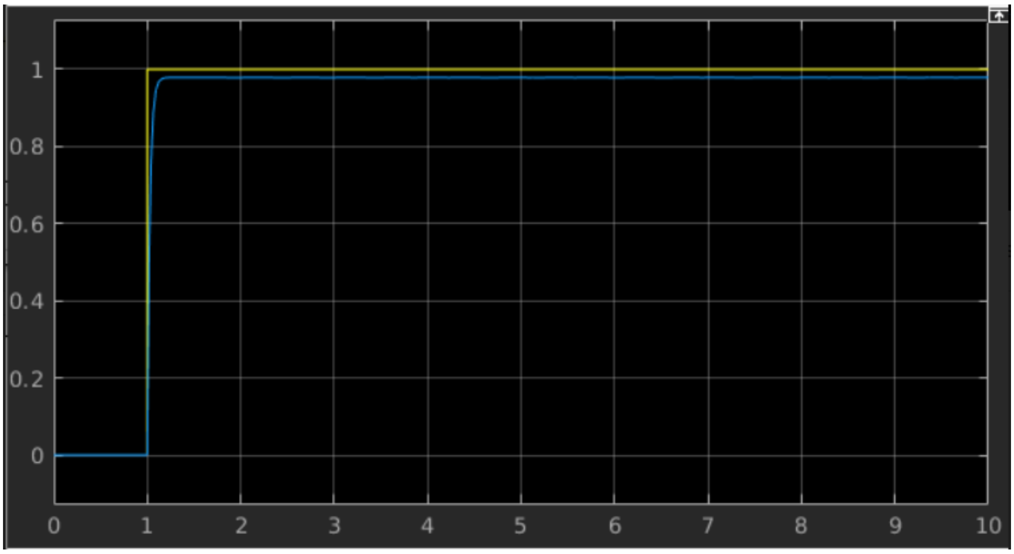


*Figure 6: step response for the velocity loop*

As shown in figure 6, the speed controller response meets the user specifications of 1.2 seconds given in table 1. Therefore, the proportional controller is good enough for speed control.

## Position control results

Using the steps mentioned in the methodology and the same steps in the above velocity design, the new plant cases $P_{new}$ were calculated as given by equation 3. Using a sampling time of T = 0.02 s and a set of frequencies

*W = 0.25,0.5,0.75,1,3,5,7,10,12,15,20,25,30,50,100,150,300,500*

The following bounds and loci were traced on the inverse Nichols chart.

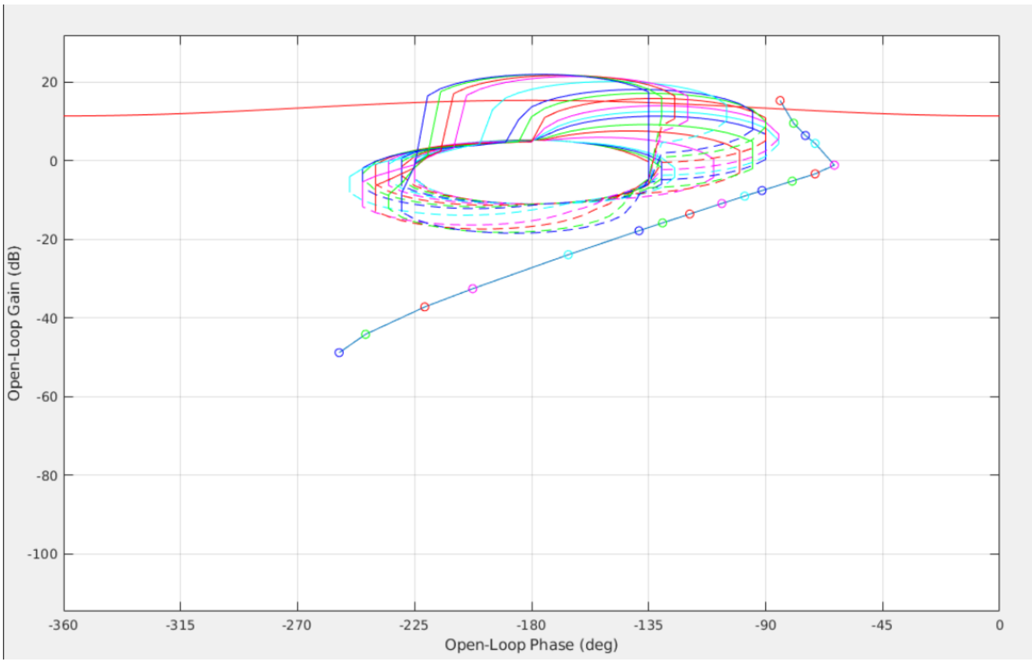


*Figure 7: Inverse Nichols chart for position control*

It was noted that the original G(s) = 1 violated the specifications and this showed that a gain alone cannot be used for a controller. The frequency points were shifted around until a valid controller for the given performance specifications and boundaries were met. The controller that satisfies both these conditions needed to have a lead/lag circuit and additional gain added to it. In addition, we had to ensure that there was a 10° margin between the frequency points on the INC to account for the margin of digital design given in the user specifications in table 1. As a result, the frequency points are carefully shifted around until the correct set of coefficients that satisfies the above specifications for the lead/lag circuit are met. the final

$$G_1(s) = \frac{149.5(s+1.635)}{s+5.506} \tag{6}$$

Again using $P_1(s)$ as our nominal plant, the block diagram that was used for the simulation is shown in figure 8 followed by the step response for position control shown in figure 9.

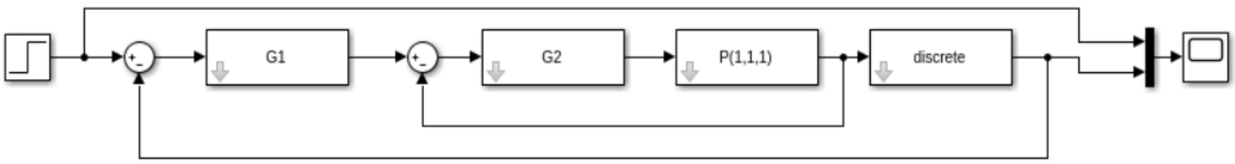


*Figure 8: Simulink block for the position control*

The discrete block accounts for the right half plane zero that estimates the discretization effects and also the ratio between the motor shaft and the position indicator. Using a period of T=0.02s the discrete block is thus a product of:

$Motor\ shaft\ to\ position\ indicator$:    $1/30s$

$Discrete\ zero:$ (1 −s(T/2)) = (1-s(0.02/2))

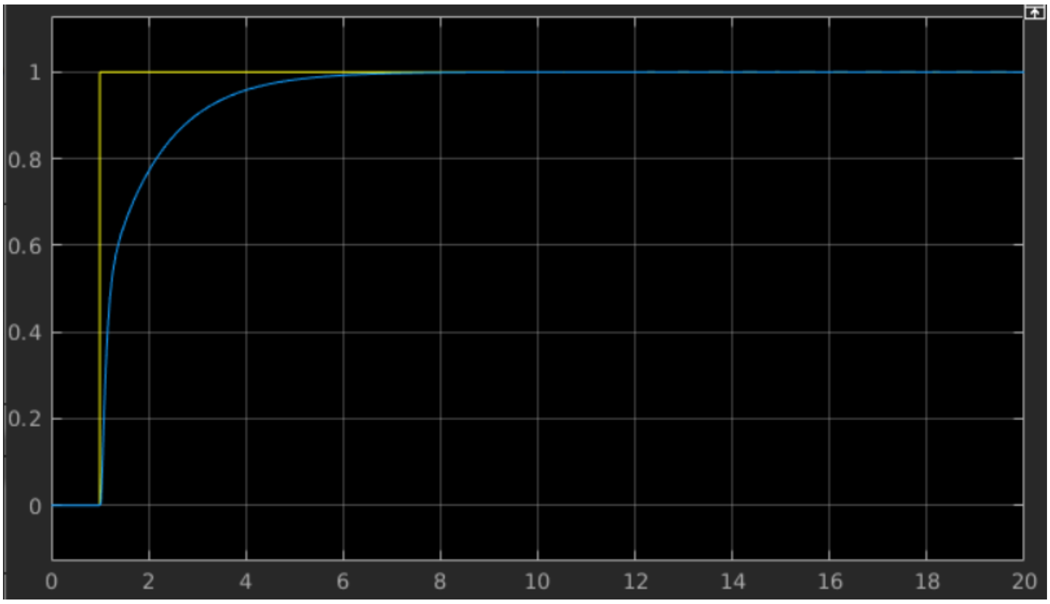


*Figure 9: Step response for position control*

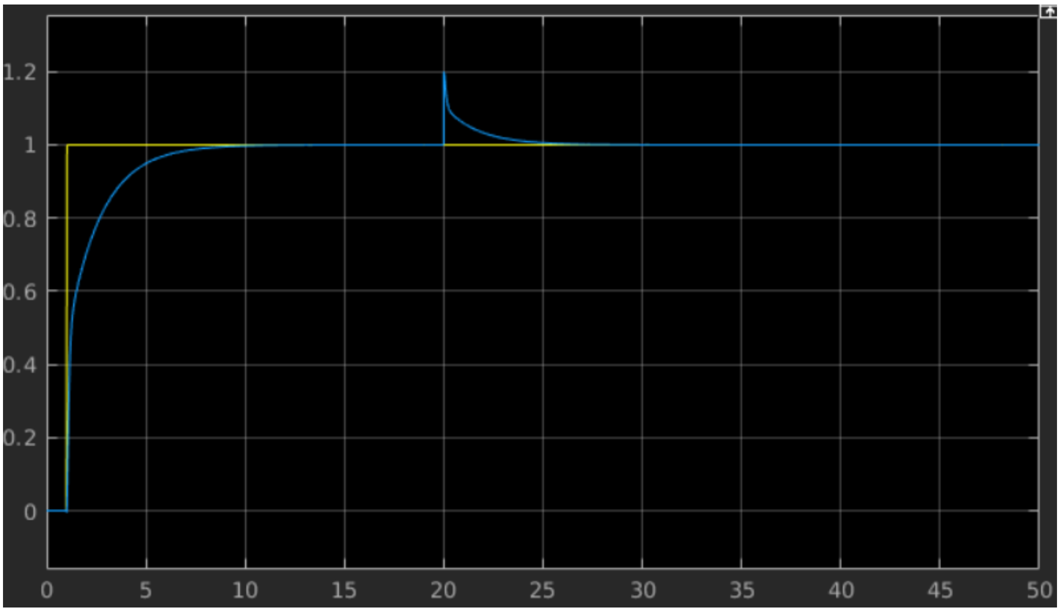


*Figure 10: Output disturbance rejection*

The above controller meets the given specifications as the overshoot is below 12.5% and the rise time is slow than 2*speed of the velocity. In addition, the controller has zero steady state error tracking thus the user specifications given in table 1 have been met.

Furthermore, figure 10 shows the behaviour of the output once a disturbance has been injected into the system. It is seen that the controller corrects the output and it goes back to tracking the set point.

# Conclusion

We have been able to design controllers that meet both speed and position control while adhering to the given user specifications. For speed control, a proportional controller of 0.6018 was good enough to meet the user specification of 1.2s for the speed response while also having and overshoot of less than 25%.

For position control, we took the digitisation into account by adding a right-half plane zero with a sampling time of 0.01s. the user specifies a 10° margin for the digital design, this is met by ensuring that there is an additional 10° between design frequencies and the resulting boundaries. After the effects of saturation had been considered, the controller that best met the specification is given in equation 6.

The overshoot margin of 12.5% for position that was originally given was met as shown in figure 7. Secondly the rise time of the controller was around 3 seconds as the step time was originally at 1 second, making it slower than the velocity response. Every given user specification has thus been met and both controllers can be used